A Bayesian approach to improving measurement precision over multiple test occasions

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MetaMetrics
Single-administration Tests

In educational measurement, there is a tendency to interpret scores from a single administration of a test as an accurate indicator of an underlying latent trait.

Usually, test scores are taken at face value, even though scores fluctuate over test occasions, e.g.

- immigration or university entrance tests
- formative assessments administered multiple-times-per-year

However, other fields make more use of prior data, or make adjustments to their estimates as new data comes to light.
Non-Farm Payroll

When the US Bureau of Labor Statistics reports this data monthly, they update the previous month, and the month before that.
Combining recent and prior scores

This paper researches a statistical approach, using Bayes theorem, to combine previous test scores with new test scores in order to arrive at a more-precise estimate of student ability.
Combining recent and prior scores

IRT scaled scores
Bayes adjusted scores

Assessment Period

Lexile
Two Approaches to estimating student ability

Familiar approach

- Raw score
  - IRT modeling
  - IRT Scaled score with standard error

Bayes approach

- IRT Scaled score + uncertainty
  - IRT Scaled score + uncertainty
  - Ability estimate + uncertainty

Prior ability estimate & recent estimate ability = new ability estimate
Steps for computation

1. Get previous values:
   Student comes into the testing session with $b_{prior}$ and $\sigma_{prior}$ from $t_0$.

2. Update values:
   At time $t_1$, student takes the test and $b_{update}$ and $\sigma_{update}$ are computed.
   
   \[ b_{update} \] takes account of learning gains since the prior estimate
   \[ \sigma_{update} \] takes account of increasing uncertainty since the prior estimate

3. Compute new values:
   Combine updated prior information with current information.
Customizable features in the model

1. Incorporating growth or learning gains
   An assumption can be made that test-takers’ skills are improving over time

2. Increasing uncertainty over time
   An assumption can be made that the more time passes between tests, the less useful prior scores are
Customizable features

1. Growth assumption
When average growth rates have been estimated for a known population, these can be incorporated into the model.

Typical Rates of Reading Growth in US Schools
1. Growth assumption
When average growth rates have been estimated for a known population, these can be incorporated into the model.

<table>
<thead>
<tr>
<th>Student grade or year</th>
<th>Average measures</th>
<th>Average annual progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd to 4th</td>
<td>664L to 803L</td>
<td>139L</td>
</tr>
<tr>
<td>4th to 5th</td>
<td>803L to 925L</td>
<td>122L</td>
</tr>
<tr>
<td>5th to 6th</td>
<td>925L to 1029L</td>
<td>104L</td>
</tr>
</tbody>
</table>

Customizable features

Growth assumption
Younger, low ability students tend to grow faster than older, more experienced students. Variable rates of growth can be modeled for specific populations.

\[ b_{update} = b_{prior} + g_s(t_1 - t_0) \]

Based on a known growth curve
Customizable features

2. Increase of uncertainty over time
   • Uncertainty from a prior test increases as time passes (i.e. uncertainty from a test taken 12 months ago is higher than a test taken 6 months ago.)
   • After how many months should we say that the information in a prior test has no value? (i.e. at what time period do we set maximum uncertainty?)

For example:

\[
\sigma_{update} = \frac{(\sigma_{maximum} - \sigma_{prior})(t_2 - t_1)}{t_{maximum}} + \sigma_{prior}
\]
Customizable features

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For example:

\[
\sigma_{update} = \frac{(\sigma_{maximum} - \sigma_{prior})(t_2 - t_1)}{1095.75} + \sigma_{prior}
\]

↑
Number of days in 3 years
Research question

To what extent do students’ reading ability estimates differ when using Rasch-scaled scores from a single administration versus Bayesian scoring with priors?

Contexts:

- L2 students in an after-school English-language instructional program in S Korea (n=25)
- L1 elementary students in schools in the US following a curriculum-based reading program (n=20,928)
Assessment instruments

Naturalistic data was obtained from two the assessment contexts
• Progress monitoring tests designed to measure reading comprehension
• Fixed-form, 30-40 multiple choice items in each test
• Specific test forms developed for different levels, as aligned with instructional program
• Scores reported on the Lexile Scale
Lexile Score Scale

Lexile measures:

- A framework for connecting readers with level-appropriate texts
- An equal interval scale, from below zero to above 2,000
- **Lexile student measures** refer to reader ability
- **Lexile text measures** refer to text complexity

770L reader

770L book
Lexile Score Scale

• Over 65 reading assessments have been linked to this scale, e.g. by seeding items from an anchor item bank into their assessments
• 100 million books, articles and websites have been measured
• Approx 35 million students receive Lexile measures globally

Scientists have made a discovery about the moon. They believe its interior contains much more water than previously thought, though exactly how much is still unclear. If scientists are correct, future astronauts may benefit from this finding. Traveling through the solar system requires extensive supplies, including water. With improved technology, astronauts could extract water from the moon and leave water from Earth off their packing lists.

The discovery could be:
- misleading
- advantageous
- symbolic
- costly
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Example: Bayes “smooths out” a student’s measures
Example: Bayes “smooths out” a student’s measures.
Example: Bayes makes very little difference
Example: Bayes reduces impact of a single poor test

- IRT scaled scores
- Bayes adjusted scores

Assessment Period:
- Fall 2014
- Spring 2015
- Fall 2015
- Spring 2016
- Fall 2016

Lexile Scale:
- 0
- 100
- 200
- 300
- 400
- 500
Example: Bayes reduces the peaks and valleys

![Line chart showing the trend of Lexile scores over assessment periods from Fall 2014 to Fall 2016, with two lines representing IRT scaled scores and Bayes adjusted scores.]
Example: Bayes reduces the peaks and valleys
Example: Bayes reduces the peaks and valleys

Bayes may be underestimating true growth velocity.

Could put a limit on prior uncertainty to increase impact of more-current tests.
Example: Bayes resists declining scores

Bayes may be overestimating true ability.
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Data Source B: Group-Level Exploration

Assessment
- Progress monitor test associated with a reading program, administered 3 times during the school year
- Specific test forms developed for each grade

Sample
- US students in general education settings

<table>
<thead>
<tr>
<th>Grade</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>311</td>
</tr>
<tr>
<td>5</td>
<td>493</td>
</tr>
<tr>
<td>6</td>
<td>4,749</td>
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<tr>
<td>7</td>
<td>4,938</td>
</tr>
<tr>
<td>8</td>
<td>4,516</td>
</tr>
<tr>
<td>9</td>
<td>3,298</td>
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<tr>
<td>10</td>
<td>1,483</td>
</tr>
<tr>
<td>11</td>
<td>765</td>
</tr>
<tr>
<td>12</td>
<td>355</td>
</tr>
<tr>
<td>Overall</td>
<td>20,928</td>
</tr>
</tbody>
</table>
The following plots compare distributions of correspondence scores (i.e. non-Bayes) with Bayes adjusted scores.
Boxplot: Correspondence: All Students (n = 20,928)

Assessment Period

Fall | Winter | Spring

Lexile

Type

Correspondence
Correspondence vs Bayes, 1\textsuperscript{st} Test Admin

* Same starting point in the Fall
Correspondence vs Bayes, 2nd Test Admin
Correspondence vs Bayes, 3rd Test Admin

Histogram of Student Ability
Type: Correspondence, Assessment Period: Spring

- Mean = 734.04
- Std. Dev. = 200.4
- N = 20,928

Histogram of Student Ability
Type: Bayes, Assessment Period: Spring

- Mean = 735.80
- Std. Dev. = 156.729
- N = 20,928
Bayesian Scoring in Repeated test-taking

Conclusions from the data
• For individuals, Bayes scoring reduces the impact of a single poor or excellent test, and reduces the peaks and valleys (more in-line with expected student change)
• Over a large sample, it creates a more normalized distribution of test scores and reduces outliers
• Bayes scoring reduces growth or declines when they occur rapidly
Discussion

Advantages to this approach

• Uses all available data to inform ability estimates, not just the most recent observation
• Provides a more precise ability estimate
• Protects against extreme single performances (both poor and excellent)
• Is sensitive to time elapsed between tests
• Supports tunable features that can control how prior and current measures are used to produce an ability estimate
Discussion

Considerations

• Difficult to explain to people
• Suitable for when you want to understand “true skill”, not for performance instances (e.g. competitions)
• Tunable features require management and monitoring
  – For example: It is possible to underestimate growth if prior uncertainty is low, and prior proficiency estimates are weighted too highly
Discussion

Considerations

• People are used to receiving a test score; however, the notion of a latent ability estimate is more nuanced.

• We are no longer answering the question “How did I do today?”; rather, we are answering “What is my updated ability estimate in light of today’s new information about me?”

• Arguably, this is a better way to evaluate examinees, than to let them re-take a test numerous times until they are lucky enough to attain a high score.
Questions

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